NOTES ON THE GRADIENT WIND IN LOW LATITUDES.

By W. J. Gibbs, M.Sc.

(Two Text-figures.)

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In the majority of meteorological text-books remarks on the gradient wind are usually restricted to the derivation and discussion of the equation involving the force due to the pressure gradient, the geostrophic force, and the cyclostrophic force, usually being expressed as the relationship

$$P = G \pm C,$$

where P is the force due to the pressure gradient,
G is the geostrophic force, and
C is the cyclostrophic force.

Because most writers are not concerned with equatorial meteorology, their studies are generally confined to higher latitudes, the equatorial regions being dismissed with the comment that there, the deflecting force due to the earth's rotation is very small and the above equation does not apply. Also, in most analyses a steady state is assumed, in which motion is at right angles to the forces and the forces are in equilibrium, and where the geostrophic force is large the assumption of this steady state is permissible; however, in lower latitudes where velocities are frequently small, the assumption of a steady state gives results which in practice are never attained.

Case at the Equator.

Consider the position at the equator where there is no deflective (Corioli's) force due to the rotation of the earth. The forces acting are the force due to the pressure gradient and, if air in motion follows a curved path, the cyclostrophic force. The cyclostrophic force is often quoted as being of major importance in low latitudes, but it is more correct to state that it is of *potential* importance, as frequently air in equatorial regions moves with a straight trajectory, and consequently the cyclostrophic force is negligible or zero, in which case the only force acting on the air is the force due to the pressure gradient.

It is obvious that air *at rest* at the equator on being controlled by a pressure gradient (the isobars being straight or curved and crossing the equator at any angle) will move in a direction from high to low pressure and *normal to the isobars.* The velocity of the air is dependent upon the force due to the pressure gradient, but is limited by the fact that the lower latitudes cover a region of converging and ascending air.

In considering the case of air *in motion* at the equator, suppose that an element of air with velocity 'V' comes under the control of a pressure gradient such as that shown in Figure 1. If the air originally has a straight line path the only force acting will be that due to the gradient, 'P'. This will tend to give the air a parabolic trajectory as shown by the dotted line, and this curved path would introduce a cyclostrophic force 'C' tending to deflect the air away from the low pressure area, as shown in the figure. It is clear that only two stable states are possible: (*a*) the case where the path becomes straight, 'C' becomes zero and air moves with direction normal to the isobars; and (*b*) the case where the isobars are cyclonic, 'P' = 'C' and wind blows in either direction around the centre of low pressure. Although theoretically possible, case (*b*) does not occur in practice mainly because of flat gradients and absence of well-developed centres of low pressure in equatorial regions.

It is evident from this brief discussion of gradients at the equator that *at the equator the direction of the gradient wind is from high to low pressure and normal to the isobars.*

General Case in Low Latitudes.

Consider now the general position in low latitudes where the geostrophic force is small.

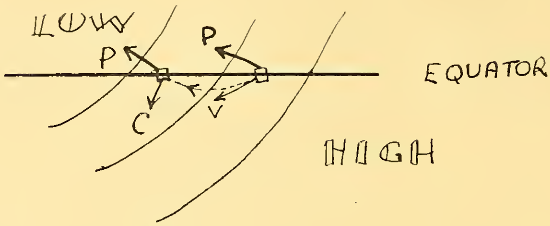

FIG. I.

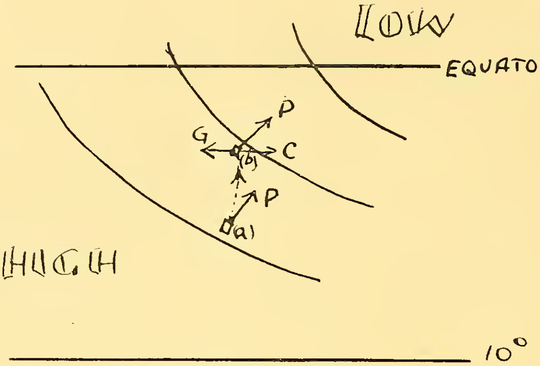

FIG. 2.

Figs. 1 and 2.

Figure 2 shows a general case of pressure distribution with a stationary element of air at 'a'. The initial movement of this element will be in the direction of the force 'P' due to the pressure gradient, but the movement will involve the geostrophic force acting at right angles to the line of motion, and so the particle will tend to a curved path as represented by the dotted line. This will introduce the cyclostrophic force so that at another position 'b', the element will have the three forces, 'P', 'G' and 'C' acting as shown in the figure. In higher latitudes the geostrophic force 'G' acting at right angles to the direction of motion soon constrains the wind to a direction parallel to the isobars, but in lower latitudes where 'G' is small, very large velocities are required to maintain a steady state.

Consider the case of straight isobars in which 'G' is equal to 'P'.

$$P = \frac{1}{\rho}\frac{dP}{dn} \qquad \text{where } \rho = \text{the density of the air}$$

$$\frac{dP}{dn} = \text{the pressure gradient.}$$

Also $G = 2wV \operatorname{Sin} \phi$ where w = angular velocity of earth

V = velocity of air

ϕ = latitude.

Using the values $\rho = 0{\cdot}0012$ gm./cc.

$w = 7{\cdot}29 \times 10^{-5}$ radians/sec.

we have the relationship $V = 5{\cdot}73 \times 10^{6} \times \frac{dP}{dn} \times \frac{1}{\operatorname{Sin} \phi}$.

Selecting common pressure gradients experienced in low latitudes, we get the following values of velocity at different latitudes:

Latitude.	Gradient of 2 millibars in 100 miles.	200 miles.	300 miles.
10°	91 m.p.h.	41 m.p.h.	30 m.p.h.
8°	112 ,,	56 ,,	37 ,,
6°	150 ,,	75 ,,	50 ,,
4°	225 ,,	112 ,,	75 ,,
2°	450 ,,	225 ,,	150 ,,

Gradient velocities greater than 30 m.p.h. are uncommon in latitudes lower than 10°, and regarding this fact in conjunction with the above table, we may deduce that for straight or anticyclonically curved isobars with the usual gradients, 'G' is never equal to 'P'. It follows that wind direction is not along the isobars, but *across* them in a direction from high to low pressure. It will be noted from the table that between latitudes 6° and 10° weak gradients give reasonable velocities, but the weakness of the gradient is such that the gradient wind is not of sufficient velocity for the development of a geostrophic force equal to the gradient force.

On the other hand, consider the case of isobars with cyclonic curvature, where, disregarding the relatively small geostrophic force, it is possible to have equilibrium between the force 'P' due to the pressure gradient and the cyclostrophic force 'C'. The value of the cyclostrophic force is given by the equation

$$C = \frac{V^2}{r} \quad \text{where } V = \text{wind velocity}$$

and r = radius of isobar along which wind blows;

so that if $C = P$

$$V^2 = \frac{r}{\rho} \cdot \frac{dP}{dn}$$

and once more, taking values of pressure gradient and radius of curvature of isobars usual in low latitudes, we get the following table of velocities:

Radius of Curvature.	Gradient of 2 millibars in 100 miles.	200 miles.	300 miles.
200 miles	40 m.p.h.	28 m.p.h.	23 m.p.h.
400 ,,	57 ,,	40 ,,	33 ,,
600 ,,	70 ,,	50 ,,	40 ,,
800 ,,	82 ,,	57 ,,	46 ,,

From this table it is seen that with weak gradients and a small radius of curvature, moderate velocities are possible and occasionally these do occur in low latitudes, but the case in which the cyclostrophic force balances the force due to the pressure gradient is not as common in equatorial regions as text-books generally indicate. This is because (*a*) radii of curvature are generally large, (*b*) well-developed low pressure systems with circular isobars are uncommon, the isobars being irregular, which prevents the steady state of balance between cyclostrophic and gradient forces being established.

The Tropical Cyclone.

The tropical cyclone, hurricane or typhoon is the only case encountered in the lower latitudes to which the formula $P = G + C$ is valid, and the absence of this phenomenon in latitudes lower than 10°S. is a sure indication of the departure of wind direction from trajectory along the isobars in that region.

Conclusion.

It is therefore apparent that in latitudes less than 10°, wind direction tends to a direction normal to the isobars from high to low pressure for straight or curved isobars.